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IDENTIFYING THE NEW KEYNESIAN PHILLIPS CURVE

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SUMMARY

Phillips curves are central to discussions of inflation dynamics and monetary policy. The hybrid new Keynesian Phillips curve (NKPC) describes how past inflation, expected future inflation, and a measure of real aggregate demand drive the current inflation rate. This paper studies the (potential) weak identification of the NKPC under Generalized Method of Moments and traces this syndrome to a lack of higher-order dynamics in exogenous variables. We employ analytic methods to understand the economics of the NKPC identification problem in the canonical three-equation, new Keynesian model. We revisit the empirical evidence for the USA, the UK, and Canada by constructing tests and confidence intervals based on the Anderson and Rubin (1949) statistic, which is robust to weak identification. We also apply the Guggenberger and Smith (2008) LM test to the underlying NKPC pricing parameters. Both tests yield little evidence of forward-looking inflation dynamics. Copyright © 2008 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Recent years have witnessed a boom in work on the Phillips curve. For a student of monetary policy and the business cycle steeped in dynamic general equilibrium methods, the revival of Phillips curve research might come as a shock. The shock might be mitigated because the Phillips curve revival features debates on the role of backward- and forward-looking expectations for inflation, on which measure of real aggregate demand most directly influences inflation, on the response of monetary policy to various disturbances, on the costs of disinflation, and on optimal monetary policy. These debates often are framed by the new Keynesian Phillips curve (NKPC) because it appears to provide a tent under which many views of inflation dynamics can exist. However, whether to be inside or outside the Phillips curve revival tent depends on the NKPC being a persuasive description of inflation dynamics.

Variations on the NKPC are just about limitless. The canonical NKPC is driven either by current real marginal cost or today's output gap and is forward-looking in the current expectation of tomorrow's inflation. Galí and Gertler (1999) added lagged inflation to create a 'hybrid NKPC', which they used to address aspects of the debate among Phillips curve revivalists. Specification of the NKPC has important implications for monetary policy, and in particular for how central banks should react to real events while maintaining inflation targets. Although contributions to this research are too numerous to list, besides Galí and Gertler (1999), Fuhrer and Moore (1995), Roberts (1995), and Sbordone (2002) made important empirical contributions. Theory and evidence about the NKPC also are reviewed by Woodford (2003).

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The hybrid NKPC is a second-order, linear, expectational difference equation. Hansen and Sargent (1980) and Sargent (1987) studied the dynamic and time series properties of this general class of stochastic models. Much empirical work on the NKPC estimates it using instrumental variables (IV) methods, as Galí and Gertler (1999) did. Generally, NKPC parameters prove difficult to pin down even with large instrument sets. This suggests weak identification. Other symptoms of this syndrome include instability of estimates across instrument sets, estimates which may approach those from ordinary least-squares and hence be inconsistent, and Wald tests with size distortions. The goals of this paper are (a) to study the economics underlying weak identification, with a view to drawing lessons and recommendations for applied work, and (b) to provide new tests of the NKPC that are robust to weak identification.

Section 2 provides background on the NKPC and begins our study of the economics of weak identification. It shows that predictability of future marginal cost or the output gap beyond that provided by current marginal cost or current or lagged inflation is necessary for identification. This predictability can be provided by higher-order dynamics in marginal cost.

Section 3 briefly considers a variety of approaches that have been proposed for dealing with the identification problem, while using standard tools of Generalized Method of Moments (GMM) estimation and testing. For example, some researchers have suggested calibrating the discount factor in the Calvo pricing model, focusing on the purely forward-looking NKPC, or indexing with lagged inflation.

Section 4 details the GMM identification problem when the hybrid NKPC is set in a typical, three-equation, new Keynesian model. We show that the hybrid NKPC cannot be identified under GMM estimation in this environment. Even persistent shocks or interest-rate smoothing in monetary policy, which are standard sources of additional dynamics, do not provide valid instruments. Thus studying the NKPC within the three-equation, new Keynesian model does not suggest additional instruments that could aid identification.

Given these analytical results, Section 5 provides tests of the NKPC that are robust to weak identification. We estimate the hybrid NKPC for the USA, UK, and Canada, using a range of instruments. We first use the Anderson and Rubin (1949) statistic to test the hybrid NKPC. This test is robust to weak or omitted instruments. Its application yields little evidence of forward-looking inflation dynamics for the USA and UK, but cannot reject the forward-looking NKPC for Canada. We also apply the methods of Guggenberger and Smith (2008), which allow more powerful tests—again robust to weak identification—of the Calvo pricing parameters that underlie the NKPC. These methods too yield little evidence in support of the hybrid NKPC for the USA, UK, and Canada.

2. BACKGROUND

A variety of pricing environments give rise to a hybrid NKPC that describes inflation, π_t :

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda x_t \tag{1}$$

where we use x_t to denote real aggregate demand (either real marginal cost or an output gap). The studies by Roberts (1997), Fuhrer and Moore (1995), and Galí and Gertler (1999) contain examples of these environments. The underlying pricing behavior can range from smooth adjustment with quadratic costs to a variation of Calvo's contract model (with or without firm-specific capital) in

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which some price-setters are backward-looking. The hybrid NKPC (1) also may be consistent with the dynamic indexing model studied by Woodford (2003) and Christiano *et al.* (2005), assuming it is written in the quasi-difference or change in inflation rather than the level.

An influential example of an environment underlying the NKPC is Calvo's pricing model. There the discount factor is β. A fraction θ of firms are not allowed to change prices each period. In Galí and Gertler's hybrid version of the model, meanwhile, a fraction ω are able to change prices but choose not to. Define $\phi = \theta + \omega[1 - \theta(1 - \beta)]$. Then the mapping between these structural parameters and the reduced-form parameters is

$$\gamma_f = \frac{\beta\theta}{\phi}, \gamma_b = \frac{\omega}{\phi}, \lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\phi} \tag{2}$$

This mapping is unique when the structural parameters are positive fractions.

It is often convenient to work with the general problem of identifying the parameters γ_f, γ_b, and λ. In this case the environment is linear, so there is no distinction between local and global identification. Throughout the paper we also assume (with one exception) that the roots of relevant difference equations imply stability and uniqueness of solutions, and that the difference equation (1) follows from a pricing model—in which all three parameters are positive—and not an observationally equivalent environment, as outlined by Beyer and Farmer (2004).

The hybrid NKPC (1) is a linear, second-order, stochastic difference equation. Our study draws on tools for formulating these problems under rational expectations developed by Hansen and Sargent (1980) and Sargent (1987). We also draw on studies of estimation in the linear-quadratic model by Gregory *et al.* (1993), West and Wilcox (1994), and Fuhrer *et al.* (1995).

GMM estimation of the hybrid NKPC (1) uses sample versions of

$$E[\pi_t - \gamma_f \pi_{t+1} - \gamma_b \pi_{t-1} - \lambda x_t | z_t] = 0, \tag{3}$$

and instruments z_t. Given moment conditions (3), a necessary condition for identification of $\{\gamma_b, \gamma_f, \lambda\}$ is that there are as many *valid* instruments as parameters (or variables that explain inflation in this linear model). A test based on over-identification requires at least four instruments or four such pieces of information. The instruments must be uncorrelated with the GMM residuals, which are essentially forecast errors. This is the *order* condition. Of course, being dated $t-1$ or earlier is not sufficient for an instrument to be valid: it must possess *incremental* information about π_{t+1}. The matrix of cross-products of the instruments and the right-hand-side variables in the hybrid NKPC cannot be singular. This is the *rank* or 'relevance' condition of IV estimation. We have omitted constant terms, as if the data have been demeaned. Of course, if in applications a constant term is included in the NKPC, a vector of ones can be used as an instrument while adding no net identifying information.

Identification obviously requires that one can predict π_{t+1} with at least one variable other than π_t, π_{t-1}, and x_t. This is a stringent requirement. Stock and Watson (1999) reported that few variables have power to forecast postwar US inflation once lagged inflation and the unemployment rate are accounted for.

But the economic structure can be used to give an alternate perspective on the potential identification problem. The second-order difference equation (1) can be rewritten in present-value

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form using the methods of Sargent (1987):

$$\pi_t = \delta_1 \pi_{t-1} + \left(\frac{\lambda}{\delta_2 \gamma_f}\right) \sum_{k=0}^{\infty} \left(\frac{1}{\delta_2}\right)^k E_t x_{t+k} \tag{4}$$

where δ_1 and δ_2 are the stable and unstable roots, respectively, of the characteristic equation:

$$-L^{-1} + \frac{1}{\gamma_f} - \frac{\gamma_b L}{\gamma_f} = 0$$

We assume that $\{x_t\}$ is of exponential order less than δ_2 so that the infinite sum in (4) is finite, and that the roots yield a unique solution to the difference equation.

Leading the present-value version (4) forward and forecasting gives $E_t \pi_{t+1}$ as

$$E_t \pi_{t+1} = \delta_1 \pi_t + \left(\frac{\lambda}{\delta_2 \gamma_f}\right) \sum_{k=0}^{\infty} \left(\frac{1}{\delta_2}\right)^k E_t x_{t+1+k} \tag{5}$$

This restatement of $E_t \pi_{t+1}$ shows that identifying γ_f requires that there be variation in the forecast of the stream $\{x_{t+1}, x_{t+2}, x_{t+3}, \cdots\}$ that is unrelated to variation in x_t and π_{t-1}. In other words, higher-order dynamics are needed for identification.

Recall that when a present-value (5) is projected on current information the number of lags is one less than the lag length in the underlying forecasting model. Thus, for example, if x_t follows an autonomous, Jth-order autoregression, then $J \geq 2$ is necessary for identification and $J \geq 3$ is necessary for over-identification with GMM. Marginal cost x_t must be predictable with at least one variable other than x_{t-1} and π_{t-1}. In particular, if x_t follows a first-order Markov process then the parameters of the second-order difference equation in inflation (1) cannot be identified by GMM. Pesaran (1987, Propositions 6.1 and 6.2) derived similar results. He observed that identifying information is available when the lag length in the process for x_t is longer than that in the difference equation.

Including the lagged, endogenous variable in predicting x may partly capture the additional information used by price-setters in forecasting. Campbell and Shiller (1987) and Boileau and Normandin (2002) developed this approach. Suppose then that the investigator predicts x with once-lagged or twice-lagged inflation in the hope of providing over-identification. Again, the lag length in the projection of the present value on current information is one less than the lag length in the forecasting equation. Recall that π_{t-1} is already in the estimating equation, so for π_{t-2} to be available as an instrument requires that x_t be predictable with π_{t-3}.

A number of researchers have used only lagged instruments in estimating (3). For example, Galí and Gertler (1999) used up to four lags of six instruments. Using lagged instruments moves the forecasting platform back in time, but does not alter the result that higher-order dynamics or additional, valid variables other than x and π are needed for identification. For future reference, denote the instrument set that excludes any current variables by z_{t-1}.

Ma (2002) was the first to raise the issue of the potential weak identification of the NKPC with GMM. He studied the US Phillips curve using the S-sets derived by Stock and Wright (2000). Mavroeidis (2004) provided a good discussion of the econometrics of weak identification and traced it to the properties of exogenous variables. He suggested using the concentration parameter as a measure of weakness of identification, but did not provide formal tests. Mavroeidis (2005) showed by simulation that standard GMM tools may be unreliable when applied to the NKPC. Dufour *et al*. (2006) simultaneously with this paper proposed and applied identification-robust tests based on Anderson and Rubin (1949) to US NKPCs measured with an output gap.

Our focus on dynamics excludes other potential sources of identification, such as structural breaks, time-varying coefficients, stochastic volatility, or the use of survey data on inflation expectations. We also do not focus on the system estimator, where additional identifying information is available from cross-equation and covariance restrictions. Systems estimation has been studied by, among others, Fuhrer and Moore (1995), Sbordone (2002, 2005), Kurmann (2005), Lindé (2005), Jondeau and Le Bihan (2003), and Fuhrer and Olivei (2004).

Our main contributions are twofold: (a) we trace the potential weak identification of the NKPC to the economic properties of the underlying new Keynesian model, and (b) we provide tests—robust to weak identification—of the forward-looking component of the NKPC for the USA, UK, and Canada both for the reduced-form parameter γ_f and for the underlying parameters of the hybrid NKPC.

3. GMM APPROACHES

The purpose of this section is to review several plausible suggestions for avoiding weak identification or non-identification and for transforming the moment condition or instruments while continuing to apply GMM, and to see whether they provide solutions. Several methods are practical, but they generally rely on specific, extra information or preclude testing the hybrid NKPC. For ease of reading, proofs of propositions are collected in Appendix A.

First, in some circumstances, the investigator may know the value of λ, either from theory or from some auxiliary statistical work. For example, if $x_t \sim I(1)$ then x_t and π_t will be cointegrated with parameter λ, which could be estimated from a static regression, as originally proposed by Engle and Granger (1987). This common, stochastic trend restriction can potentially aid identification of the remaining parameters, γ_f and γ_b.

Proposition 1 If a consistent estimate $\hat{\lambda}$ is available, then an additional instrument is available in z_t but not in z_{t-1}.

The logic behind this proposition is simply that if λ is known then x_t becomes available as an instrument to help forecast π_{t+1}. But with a lagged instrument set z_{t-1}, three variables in the NKPC (3) remain to be forecast, $\{\pi_{t+1}, \pi_t, x_t\}$, even given an estimate $\hat{\lambda}$. The last part of Proposition 1 is a generalization of an example found in Gregory *et al.* (1993), who modelled x_t as a random walk. According to Gregory *et al.*, lagged instruments could not identify the parameters of the difference equation without higher-order dynamics in the x-process. Proposition 1 also is relevant to price-setting rules that are written in terms of the level of prices, rather than the inflation rate, because the price level is more likely to be nonstationary yet cointegrated with the fundamental.

Proposition 2 Restricting $\gamma_b = 0$, or $\gamma_b = 1 - \gamma_f$, or calibrating a discount factor β in an underlying pricing model may aid identification.

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First, if the investigator imposes $\gamma_b = 0$, so that the NKPC is purely forward-looking then the variable π_{t-1} is now free to play the role of an instrument for π_{t+1}. Mavroeidis (2004, 2005) provided a detailed discussion of this case.

Second, a number of authors suggest imposing the restriction $\gamma_f + \gamma_b = 1$. This restriction means that there is no long-run trade-off between inflation and real activity in levels. Lindé (2005) and Rudd and Whelan (2006) studied this restricted model with systems estimators. Given this restriction, the NKPC becomes

$$\Delta\pi_t = \tilde{\gamma}_f E_t \Delta\pi_{t+1} + \tilde{\lambda} x_t \tag{6}$$

where $\tilde{\gamma}_f = (1 - \gamma_b)/\gamma_b$ and $\tilde{\lambda} = \lambda/\gamma_b$. Christiano *et al.* (2005) and Woodford (2003) showed that this revised Phillips curve (6) is implied by a staggered pricing mechanism in which firms cannot commit to a new price but instead set their price at date t by adding lagged, aggregate inflation to fully index their previous period's price. The restriction also holds approximately in the Galí–Gertler model for plausible values of β. The rewriting (6) again shows that π_{t-1} again is now eligible as an instrument. (As an aside, we note that one might well find $\tilde{\gamma}_f > 1$ in estimating the transformed Phillips curve equation (6), for the formula for $\tilde{\gamma}_f$ shows that such values would be implied by any $\gamma_b < 0.5$.)

Third, the NKPC sometimes is viewed as stemming from an underlying Calvo pricing model, with three deep parameters: β, a discount factor, the fraction of firms able to change price, and the fraction able to change price that do not. Pre-setting β amounts to setting γ_f conditional on γ_b and λ, and so it may allow identification.

An important corollary of Proposition 2, however, is that naturally none of the restrictions in Proposition 2 can be tested using over-identifying information if the restriction is necessary for identification so that the two remaining parameters are just identified. For example, an investigator who achieved identification by imposing the first restriction, $\gamma_b = 0$, could not test the hybrid NKPC against the purely forward-looking one. That is because the unrestricted, hybrid model then would not be identified.

As an interesting way to provide evidence on the hybrid NKPC, Rudd and Whelan (2005), Galí *et al.* (2005), and Guay *et al.* (2003) solved the hybrid NKPC difference equation forward, but truncated after K leads. Rudd and Whelan (2005) motivated instrumenting the present discounted value of x instead of π_{t+1} by the possibility of specification error. This led them to estimate by instrumental variables

$$E\left[\pi_t - \delta_1\pi_{t-1} - \frac{\lambda}{\delta_2\gamma_f}\sum_{k=0}^{K}\delta_2^{-k}x_{t+k}|z_{t-1}\right] \tag{7}$$

Proposition 3 Solving the NKPC forward and truncating provides no additional information to aid identification (or improve efficiency).

This proposition simply reflects the fact that the transformation or forward solution in the estimating equation still involves the three parameters $\{\gamma_f, \gamma_b, \lambda\}$ without affecting the number of relevant instruments.

In GMM estimation it is sometimes useful to use lagged residuals as instruments. For example, they have helpful scaling properties, for they are of the same order of magnitude as the residuals that are being minimized in estimation. The next result shows that this device is unavailable for the NKPC.

Proposition 4 Whether z_t or z_{t-1} is adopted, the GMM residual is a MA(1) process, so any instrument set must exclude once-lagged GMM residuals.

The GMM residual includes a two-step forecast error, which naturally follows a first-order moving average spanning periods $t+1$ and t. Thus the residual is correlated with the lagged residual, which violates the orthogonality condition. However, this moving average can be accounted for in constructing the weighting matrix in GMM estimation.

In summary, several methods suggested to ward off weak identification are practical, but they generally rule out testing of the hybrid NKPC. Thus identification and testing requires higher-order dynamics or additional variables to predict inflation. A natural question concerns the economic interpretation of these features. The next section looks at their availability in a new Keynesian economic model.

4. ECONOMIC SOURCES OF WEAK IDENTIFICATION

Up to this point, we have noted that identifying the hybrid NKPC depends on the multi-step predictability of the x-process. However, real marginal cost or the output gap is endogenous in a dynamic, stochastic, general-equilibrium model. We study identification in a more complete model in this section. It seems natural to work with a typical, new Keynesian, trinity (i.e., three-equation) model (NKTM) consisting of an NKPC, a linearized, dynamic IS schedule, and a Taylor rule. Let y be the output gap and R be the interest rate set by the central bank (the nominal federal funds rate in the USA). The system is

$$\begin{aligned} \pi_t &= \gamma_f E_t\pi_{t+1} + \gamma_b\pi_{t-1} + \lambda y_t + \varepsilon_{\pi t} \\ y_t &= \beta_f E_t y_{t+1} + \beta_b y_{t-1} - \beta_R(R_t - E_t\pi_{t+1}) + \varepsilon_{yt} \\ R_t &= \omega_\pi\pi_t + \omega_y y_t + \varepsilon_{Rt} \end{aligned} \tag{8}$$

Our focus in this paper is on estimating the hybrid NKPC by replacing $E_t\pi_{t+1}$. The NKTM (8) automatically yields such forecasts, which we next derive. Using the policy rule to replace the interest rate in the equations for inflation and the output gap gives

$$\pi_t = \gamma_f E_t\pi_{t+1} + \gamma_b\pi_{t-1} + \lambda y_t + \varepsilon_{\pi t} \tag{9}$$
$$y_t = \beta_f\varphi E_t y_{t+1} + \beta_R\varphi E_t\pi_{t+1} + \beta_b\varphi y_{t-1} - \beta_R\omega_\pi\varphi\pi_t + \varphi(\varepsilon_{yt} - \beta_R\varepsilon_{Rt})$$

where

$$\varphi \equiv (1 + \beta_R\omega_y)^{-1} \tag{10}$$

Stack the endogenous variables as $w_t = (\pi_t\ \ y_t)'$, and the corresponding, composite shocks as $\varepsilon_t = (\varepsilon_{\pi t}\ \ \varphi(\varepsilon_{yt} - \beta_R\varepsilon_{Rt}))'$. This allows us to write the system (9) as

$$[I - f]w_t = cE_t w_{t+1} + dw_{t-1} + \varepsilon_t, \tag{11}$$

where the 2×2 matrices of the system of second-order difference equations (11) are

$$c = \begin{pmatrix} \gamma_f & 0 \\ \beta_R\varphi & \beta_f\varphi \end{pmatrix}, \quad d = \begin{pmatrix} \gamma_b & 0 \\ 0 & \beta_b\varphi \end{pmatrix},$$

and f has zeros on the diagonal:

$$f = \begin{pmatrix} 0 & \lambda \\ -\beta_R\omega_\pi & 0 \end{pmatrix}.$$

The bivariate system (11) can be rearranged this way:

$$cE_t w_{t+1} - [I - f]w_t + dw_{t-1} = -\varepsilon_t. \tag{12}$$

We assume uniqueness and stability, and specifically that $\omega_\pi > 1$. This restriction on monetary policy satisfies the well-known Taylor principle. Under this restriction, only fundamental shocks, ε_t, drive inflation and the output gap. The unique solution takes a first-order form:

$$w_t = aw_{t-1} + b\varepsilon_t \tag{13}$$

where a and b are 2×2 matrices. The solution (13) is the equilibrium vector process of the new Keynesian economy (8). Solving for a and b by guess-and-verify methods leads to a system of polynomials in the lag operator. Factoring a multivariate spectral density matrix usually requires numerical methods; a and b cannot be found analytically in general. For discussion and examples, see Hansen and Sargent (1981), Zadrozny (1998), and Sayed and Kailath (2001). Nonetheless, the form of the solution (13) tells us much about the necessary conditions for identification.

Proposition 5 In the new Keynesian, three-equation model with unpredictable shocks, the hybrid NKPC cannot be identified by single-equation GMM.

The result follows from the first-order Markov nature of w_t. With y_t and π_{t-1} already entering the hybrid NKPC, there are no further variables available to instrument for π_{t+1} in GMM estimation.

There will be higher-order dynamics in the *univariate* time series process for y_t implied by the NKTM. Marginalizing the VAR gives

$$y_t = a_{22}y_{t-1} + a_{21}a_{12}\sum_{j=0}^{\infty} a_{11}^j y_{t-2-j} \tag{14}$$

But there is no additional information in the lagged values of y beyond that contained in π_{t-1}. Thus, finding higher-order dynamics in y is necessary, but not sufficient for identification in GMM. Although the NKTM can produce higher-order output dynamics, as in (14), these do not yield relevant instruments. Lagged inflation already enters the hybrid NKPC. Proposition 5 implies that identifying the NKPC must rely on cross-equation restrictions in this system, as adopted by Lindé (2005), for example.

Persistent shocks are a potential source of identifying information. For identification of the NKPC, we require three instruments that jointly provide information on each of the regressors $E_t\pi_{t+1}$, π_{t-1}, and y_t and are uncorrelated with the residual $\varepsilon_{\pi t}$. Combining persistence in shocks with the endogenous persistence in the NKTM yields higher-order dynamics. But it turns out that these do not provide identifying information.

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Proposition 6 In the NKTM (8) with persistent shocks the hybrid NKPC is not identified under GMM.

The shocks are unobservable so their persistence adds persistence to the endogenous variables. For example, adding one or more AR(1) shocks makes the endogenous variables follow an AR(2) system. These lagged, endogenous variables may help predict π_{t+1}, but they are not valid instruments because they are correlated with the GMM residual $\varepsilon_{\pi t}$. The proof shows that this endogeneity arises whether the persistence originates in the NKPC or in the other equations of the NKTM. Lagging the observable variables enough to make them uncorrelated with $\varepsilon_{\pi t}$ also makes them unrelated to π_{t+1}. Section 2 showed that higher-order dynamics can provide identification if they are exogenous and observable. But the NKTM creates endogenous dynamics.

Lagged variables are not valid as instruments when there are persistent shocks, because the shocks also are cross-correlated. Calibration of the NKTM provides a numerical example. Let $\gamma_f = 0.3$, $\gamma_b = 0.6$, $\lambda = 0.025$, $\beta_f = 0.2$, $\beta_b = 0.6$, $\beta_R = 0.5$, $\omega_\pi = 1.5$, and $\omega_y = 0.25$. Suppose that the two shocks in ε_t follow uncorrelated, first-order autoregressions, with coefficients 0.95. Under this calibration, numerical guess-and-verify gives

$$a = \begin{pmatrix} 0.7727 & 0.0254 \\ -0.1970 & 0.5979 \end{pmatrix}$$

and

$$b = \begin{pmatrix} 2.1089 & 0.0964 \\ 0.5831 & 1.4281 \end{pmatrix}$$

This numerical example shows that a reasonable calibration of the NKTM gives an equilibrium law of motion (13) with correlated, reduced-form shocks even when the NKTM structural shocks are uncorrelated.

What if $\gamma_b = 0$ so that we have the purely forward-looking NKPC? In that case a persistent shock makes π_{t-1} help predict π_{t+1}, as Campbell and Shiller (1987) and Boileau and Normandin (2002) noted. With π_{t-1} excluded from the NKPC the lagged inflation rate now is available as an instrument. However, it also is correlated with the residual $\varepsilon_{\pi t}$ when the shocks are persistent, and so is invalid.

Shock persistence translates into serial correlation in inflation and the output gap. This finding may help to explain the long lags in estimated NKTM inflation and output gap equations reported, for example, by Lindé (2005) and Jondeau and Le Bihan (2003). Likewise, Ireland (2004) found that an autocorrelated cost shock in the NKPC was necessary for empirical success of the NKTM. But systems estimators that exploit cross-equation restrictions are necessary to identify the NKPC, at least in the NKTM environment.

Persistence in policy interest rates often is attributed not to persistence in the shock but to a lagged, dependent variable in the policy rule, i.e., to interest-rate smoothing. Suppose the policy rule is

$$R_t = (1 - \upsilon)(\omega_\pi \pi_t + \omega_y y_t) + \upsilon R_{t-1} + \varepsilon_{Rt} \tag{15}$$

with $0 < \upsilon < 1$. This is another standard modification to the NKTM. We next show that, once again, this feature provides dynamics but does not allow the investigator to identify the NKPC using GMM.

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Proposition 7 In the NKTM (8) with interest-rate smoothing in monetary policy the hybrid NKPC is not identified under GMM.

The logic behind this result is simple. The policy rule (15) is Markov so the entire NKTM remains Markov. Thus only the current interest rate, R_t, is newly available as an instrument with which to forecast π_{t+1}. But current inflation, π_t, enters the policy rule, so R_t is correlated with $\varepsilon_{\pi t}$ and so is invalid as an instrument.

There is an analogous, negative result on identification when the NKTM (8) possesses multiple equilibria. Lubik and Schorfheide (2004) study a NKTM that associates the indeterminacy with passive monetary policy, $\omega_\pi < 1$, and sunspot (i.e., extrinsic) shocks. Under $\omega_\pi < 1$, they show that the rational expectations forecast of π_t and y_t is a first-order VAR with forecast innovations a function of the fundamental shocks ε_t and the rational expectation forecast errors, ϕ_t:

$$[I - \tau_w L]E_t w_{t+1} = \tau_\varepsilon \varepsilon_t + \tau_\phi \phi_t \tag{16}$$

where $\phi_{t+1} = [y_{t+1} - E_t y_{t+1} \ \pi_{t+1} - E_t \pi_{t+1}]'$ and the τ matrices are functions of the parameters of the NKTM. Given the linear NKTM (8), this class of passive monetary policies also permits ϕ_t to be a linear function of ε_t and a vector of sunspot shocks, ψ_t. It follows from these facts—$E_t w_{t+1}$ is the VAR(1) (16) and ϕ_t depends on ψ_t, besides fundamental shocks—that w_t becomes a (restricted) bivariate ARMA process rather than a pure bivariate autoregression:

$$[I - \mu L]w_t = \kappa_\vartheta [I - \mu\theta_\vartheta L]\vartheta_t + \kappa_\psi [I - \mu\theta_\psi L]\psi_t \tag{17}$$

where μ denotes the stable eigenvalue of (16) and the κ and θ matrices are functions of the NKTM parameters. Note that the first-order moving average of the bivariate ARMA process (17) is a function of the fundamental and sunspot shocks. The econometrician focuses on the sunspot to connect the observed data to one of the multiple equilibria. This motivates Lubik and Schorfheide to argue that the sunspot shock interpretation of indeterminacy (created by $\omega_\pi < 1$) explains serially correlated inflation and output gap data.

Proposition 8 When the NKTM (8) possesses multiple equilibria and the rational expectations forecast errors are a (linear) function of the fundamental and extrinsic shocks, the hybrid NKPC is not identified under GMM.

The key to Proposition 8 is that the lack of restrictions on the rational expectations forecast errors under indeterminacy means that there is no additional identification information. Although fundamental and sunspot shocks are news for an econometrician attempting to estimate the NKTM (8), these shocks do not help forecast π_{t+1}. However, this approach to identifying the NKPC within a larger model imposes persistence and cross-equation restrictions on the forecast innovation of the bivariate ARMA process (17) of y_t and π_t, which can yield additional information for identification in the system.

This section has focused on potential for the NKTM to allow estimation of the hybrid NKPC when using instrumental variables. The main finding is that the NKPC cannot be identified by GMM in the NKTM environment, even if there are persistent shocks, interest-rate smoothing, or sunspot equilibria.

5. REVISITING THE EVIDENCE: ROBUST TESTS

Finding that the NKPC is not identified under GMM in the new Keynesian model leads us to consider tests that preserve the limited information features of GMM but are robust to weak identification. In this section we first estimate hybrid NKPCs for the USA, UK, and Canada. Using lagged values of inflation and marginal cost as instruments does not lead to singularity (as the NKTM would predict) but does suggest that identification is weak. We then draw inferences without using a system by using statistics from Anderson and Rubin (1949) plus some recent developments. The data consists of GDP inflation and measures of real marginal cost. Appendix B describes the data sources, while Figure 1 provides time-series plots for the three countries.

5.1. Statistics

First, we study the time-series properties of x_t. We estimate univariate autoregressions for x_t, and test the lag length from $J = 1$ to $J = 6$ lags using a likelihood ratio statistic, the AIC, and the SIC. Recall from Section 2 that—if there are no instruments other than lags of x—then $J \geq 2$ is necessary for identification in GMM. We next include lagged values of inflation and report the results of a pre-test of the null hypothesis that $\{\pi_t\}$ does not Granger-cause $\{x_t\}$. Section 2 also noted that finding a role for lagged inflation in forecasting x suggests that further instruments may be available. These could include lags of inflation beyond the first two or other variables that lead to Granger-causality because of the superior information of price-setters.

Second, our main interest is in instrumental-variables estimation, so we estimate

$$E[\pi_t - \gamma_f \pi_{t+1} - \gamma_b \pi_{t-1} - \lambda x_t | z_t] = 0 \tag{18}$$

by GMM and report point estimates and standard errors as well as the J-test statistic of over-identifying restrictions and its p-value. Following Proposition 4, GMM estimators will allow for a first-order moving average in the GMM residual. The weighting matrix will be the continuous-updating version introduced by Hansen *et al.* (1996), which has good finite-sample properties and is invariant to the normalization of the hybrid NKPC (1). We also include results with z_{t-1}, to allow for the possibility that z_t is correlated with the GMM residual due to time aggregation or measurement error. Of course, these standard estimates and inferences may be suspect due to weak identification but the idea is to show how conclusions may differ between these methods and those that are robust to weak identification.

Third, we calculate Anderson–Rubin (1949) statistics to test several hypotheses, and find the implied confidence intervals. Excellent surveys of inference under weak identification are provided by Dufour (2003) and Andrews and Stock (2007). For the just-identified case, the Anderson–Rubin test is preferred, according to these authors. The statistics from GMM estimation (18) depend on nuisance parameters under weak identification. In contrast, the AR statistics are pivotal in finite samples. To test H_0: $\gamma_f = \gamma_{f0}$ one projects as follows:

$$\pi_t - \gamma_{f0} \pi_{t+1} = \alpha_0 + \alpha_1 \pi_{t-1} + \alpha_2 x_t + \alpha_3 u_t \tag{19}$$

with auxiliary variables u_t, then constructs the Anderson–Rubin (AR) F-statistic for H_0': $\alpha_3 = 0$. The idea is that there should be no further role for u_t at the true value for γ_f. In our case, γ_f is a scalar. This yields an $F(k, T - k - 2)$ statistic, where $k + 2$ is the total number of exogenous

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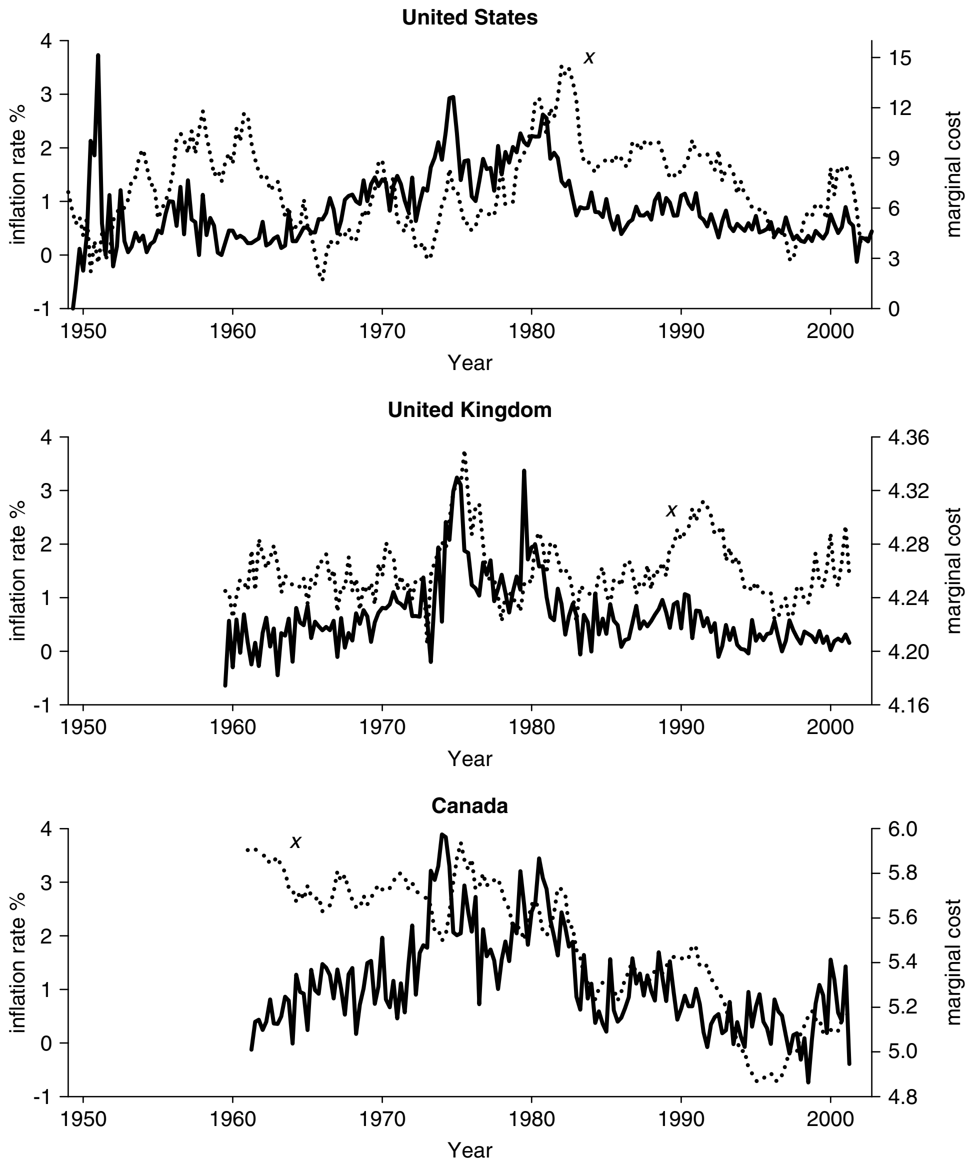

Figure 1. Inflation and marginal cost

variables and instruments. The AR statistic provides a test which is robust to (a) weak instruments and (b) omitted instruments. We do not need all the u-elements necessarily, but power is lower if irrelevant instruments are included. The test statistic also is robust to misspecification of the forecasting rule for π_{t+1} (i.e., its size is not affected, though again its power may be).

The distributional assumption underlying the statistic's being pivotal in finite samples is normality of the GMM residuals. In the literature, the main drawbacks to this approach arise

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when the structural equation is nonlinear, or when there is more than one endogenous, explanatory variable and we want to study subsets of their coefficients. But here the hybrid NKPC is linear, we begin by treating x_t as exogenous, and γ_f is a scalar.

The AR statistics also can be used to construct confidence intervals. A confidence set is

$$C(\alpha) = \{\gamma_{f0} : AR(\gamma_{f0}) \leq F_\alpha(k, T - k - 2)\} \tag{20}$$

Since γ_f is a scalar, there is a quadratic solution, given by Zivot *et al.* (1998). The coefficients of the quadratic equation are functions of the data and the F-statistic at significance level α and degrees of freedom dim(u) and $T - 2 - k$. With over-identification this confidence set can be empty. Without identification, it can be unbounded. More generally, Andrews and Stock (2007) described how this confidence interval may be conservative (too wide) if there are other endogenous right-hand-side variables.

Fourth, one criticism of the AR tests is that they lack power when there is over-identification. We find these tests reject for the US and UK data. Thus, a lack of power is not a a central issue in this study. A variety of approaches have been proposed recently to correct this potential shortfall. We apply the LM test of Guggenberger and Smith (2008), which is robust to weak instruments and suffers no power loss as over-identification rises. It also allows tests with multiple endogenous variables. Guggenberger and Smith (GS) presented Monte Carlo work in which their LM statistic has good sampling properties. For example, its power properties are comparable to the test of Kleibergen (2005), and sometimes superior when there are many instruments. Thus our evidence complements Dufour *et al.* (2006), who applied Kleibergen's test to the US NKPC.

A further advantage of the GS LM statistic is that it allows tests of parameters from nonlinear moment condition models. Recall that the NKPC interprets the parameters $\{\gamma_f, \gamma_b, \lambda\}$ as arising from an underlying structure with parameters $\{\beta, \theta, \omega\}$ given the mapping (2). We employ the GS LM statistic to test hypotheses on several values of θ and ω. In this approach we assume that parameters not under test are strongly identified. In test statistics, these parameters are replaced by consistent estimates. For methods to test subsets of parameters under weak identification, also see Dufour and Taamouti (2005), Guggenberger and Smith (2008, Section 2.3), and Section 7.8 of Andrews and Stock (2007).

5.2. United States

Table I presents evidence on real marginal cost dynamics for a US sample of 1949Q1–2001Q4. We fail to reject the null that inflation does not Granger-cause real marginal cost according to the first two rows of Table I. In addition, the AIC and LR statistics choose a lag length of 3, while the SIC selects a lag length of 1 for the x-autoregression. It is not surprising then that the second-order coefficient in this regression is insignificantly different from zero. The implication of these pre-tests is that finding relevant instruments may be a challenge in the US data. Although US real marginal cost is persistent (the half-life of a shock is about seven quarters), there is not strong evidence of higher-order dynamics in US real marginal cost. Campbell and Shiller (1987) and Boileau and Normandin (2002) also showed that the presence of other predictors of x_t should lead to a role for lagged inflation, yet we find none here. Thus, the quest for other instruments may not be fruitful.

Table II contains single-equation GMM estimates. Most of the work is done by the instruments $\{\pi_{t-1}, x_t, x_{t-2}\}$, as is suggested by the pre-test evidence that only x_t and x_{t-2} help forecast x_{t+1}.

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Table I. Granger non-causality tests

Country	Lag length (d.f.)	$p\pi \not\mapsto x$	$px \not\mapsto \pi$
USA	3	0.18	0.05
USA	4	0.24	0.08
UK	4	0.01	0.00
UK	5	0.01	0.00
Canada	3	0.00	0.73
Canada	4	0.00	0.63

The lag lengths, $\hat{J}$, are the same as those selected by information criteria. Entries are p-values for the null hypothesis that the first variable does not Granger-cause the second variable. Data sources and sample sizes are given in Appendix B.

Adding further instruments increases the precision slightly but does not lead to significant changes in the estimates. Omitting x_t from the instrument set does have an effect though, as it leads to $\hat{\gamma}_f > 1$ and $\hat{\gamma}_b < 0$, for example. The associated J-test clearly does not reject the over-identifying restrictions.

Whenever we include x_t in z_t, the estimated weight on lagged inflation, $\hat{\gamma}_b$, ranges from 0.28 to 0.42, depending on the instrument set. The GMM estimates show these expectations are dominated by forward-looking expectations because $\hat{\gamma}_f$ ranges from 0.52 to 0.70. The response of π_t to x_t, denoted $\hat{\lambda}$, also takes plausible values, between 0.1% and 0.9%, but is not statistically significant (for a 5% test). Our results are comparable to those of Galí and Gertler (1999, Table I), but we obtain smaller and insignificant estimates of λ using smaller instrument sets.

Table II also includes estimates of $\{\theta, \omega, \beta\}$ from the US data. The estimates imply that about 10% of firms have the opportunity to change price each quarter and of those 30–55% choose not to do so. The point estimates $\hat{\beta}$ are implausibly low for quarterly data, but all within one

Table II. US new Keynesian Phillips curve

$$E[\pi_t - \gamma_f E_t \pi_{t+1} - \gamma_b \pi_{t-1} - \lambda x_t | z_t] = 0$$

$1949Q1$–$2001Q4 \quad T = 212$

Instruments	$\hat{\gamma}_f$ (SE)	$\hat{\gamma}_b$ (SE)	$\hat{\lambda}$ (SE)	$\hat{\omega}$ (SE)	$\hat{\theta}$ (SE)	$\hat{\beta}$ (SE)	χ^2 (d.f.) (p)
π_{t-1}, x_t, x_{t-2}	0.685	0.300	0.001	0.406	0.961	0.965	—
	(0.357)	(0.247)	(0.007)	(0.465)	(0.163)	(0.306)	
$\pi_{t-1}, x_t, \cdots, x_{t-2}$	0.527	0.415	0.009	0.566	0.902	0.797	2.11(1)
	(0.298)	(0.205)	(0.005)	(0.321)	(0.094)	(0.480)	(0.34)
$\pi_{t-1}, x_t, \cdots, x_{t-4}$	0.706	0.275	0.008	0.333	0.892	0.961	3.47(3)
	(0.223)	(0.158)	(0.006)	(0.263)	(0.064)	(0.169)	(0.48)
$\pi_{t-1}, \pi_{t-2}, x_t, \cdots, x_{t-4}$	0.701	0.278	0.009	0.338	0.893	0.956	3.48(4)
	(0.188)	(0.141)	(0.005)	(0.234)	(0.054)	(0.135)	(0.63)
$\pi_{t-1}, x_{t-1}, \cdots, x_{t-4}$	1.288	−0.138	0.020	−0.099	0.826	1.118	0.63(2)
	(0.806)	(0.570)	(0.023)	(0.349)	(0.059)	(0.113)	(0.88)
OLS	0.461	0.456	−0.001	0.655	1.008	0.657	
	(0.049)	(0.052)	(0.007)	(0.023)	(0.009)	(0.008)	

The estimation sample runs from $1949Q1$ to $2001Q4$, based on the complete $1947Q1$–$2002Q2$ sample. Tests of the over-identifying restrictions use the J-statistic.

standard error of plausible values close to 1 (again with the exception of the case with only lagged instruments).

Next, we provide tests for the forward-looking component in the US NKPC that are robust to weak identification. Table III presents AR F-statistics and their associated p-values based on equation (19) and a grid of potentially 'true' $\gamma_f = \gamma_{f0}$. We set γ_{f0} to [0.0, 0.2, 0.5, 0.6, 0.7, 0.8, 0.9, 0.99]. The AR statistics in the first row reveal little evidence against the null of $\gamma_f = \gamma_{f0}$, for any of these values of γ_{f0} given $u_t = x_{t-2}$. When we add instruments though—in the next two rows—we can reject any of the null hypotheses at standard significance levels. Thus, lags of real marginal cost besides x_{t-2} matter for predicting the quasi-difference of π_t and π_{t+1}. The test is correctly sized even if these added instruments are weak, which gives us a formal rejection of the forward-looking model.

Table III also provides information on the asymptotic 95% confidence interval $C(\alpha = 0.05)$ of γ_f, given in (20). In the first row, with $u_t = x_{t-2}$ the confidence interval includes all values of γ_f between 0 and 1 (as well as the corresponding GMM estimate from Table II). Wide confidence intervals reflect weak identification just as do the large S-sets found by Ma (2002). But when we add instruments in the second and third rows the corresponding 95% confidence intervals are empty. These findings constitute evidence against the hybrid NKPC.

It is interesting to note the contrast with some of the findings of Dufour *et al.* (2006), who used a real-time, linearly detrended, output gap measure in the US NKPC and could not reject a significant, forward-looking component in US inflation using robust methods. It remains an open question how sensitive test results are to the choice between using marginal cost or the output gap in the NKPC or, indeed, to different ways of measuring these variables.

Table IV presents the GS test statistics for various values of θ and, separately, of ω. The grids for these underlying parameters are suggested by the GMM estimates in Table II. Across a gird of values, the test rejects each value of θ with the exception of (a) $\theta = 0.9$ on the just-identified instrument set (which yields an estimate of β greater than one) and (b) $\theta = 0.8$ with the lagged instrument set (which yields an estimate of β greater than one and an estimate of ω less than zero). In the lower panel of Table IV we find that, for those ω for which the GS test does not reject (at conventional levels of significance), the estimates of θ are between 0.93 and 1.03. Joint tests (not shown but available on request) lead to similar conclusions. With the largest instrument sets, the test also rejects each value of ω.

Table III. US NKPC: Tests of $H_0 : \gamma_f = \gamma_{f0}$

$$\pi_t - \gamma_{f0}\pi_{t+1} = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2 x_t + \alpha_3 u_t$$

Anderson–Rubin statistic

$1949Q1–2001Q4 \quad T = 212$

γ_{f0} =	0.00 (p)	0.20 (p)	0.50 (p)	0.60 (p)	0.70 (p)	0.80 (p)	0.90 (p)	0.99 (p)
$u_t =$								
$\{x_{t-2}\}$	2.15 (0.14)	1.31 (0.25)	0.21 (0.65)	0.04 (0.83)	0.00 (0.97)	0.07 (0.80)	0.21 (0.64)	0.39 (0.53)
$\{x_{t-1}, x_{t-2}\}$	4.43 (0.01)	5.17 (0.01)	5.85 (0.00)	5.83 (0.00)	5.68 (0.00)	5.45 (0.00)	5.17 (0.01)	4.90 (0.01)
$\{x_{t-1}, \ldots, x_{t-4}\}$	2.47 (0.05)	2.92 (0.02)	3.34 (0.01)	3.33 (0.01)	3.24 (0.01)	3.10 (0.02)	2.93 (0.02)	2.77 (0.03)

The Anderson–Rubin statistics are based on equation (19).

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Table IV. US NKPC: Tests of $H_0 : \theta = \theta_0$ or $H_0 : \omega = \omega_0$

$$E[\gamma_f \pi_{t+1} - \pi_t + \gamma_b \pi_{t-1} + \lambda x_t | z_t] = 0,$$

$$\gamma_f = \frac{\beta\theta}{\phi}, \quad \gamma_b = \frac{\omega}{\phi}, \quad \lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\phi}, \quad \phi = \theta + \omega[1 - \theta(1-\beta)]$$

Guggenberger–Smith LM statistic $\sim \chi^2(1)$

$1949Q1$–$2001Q4$ $T = 212$

θ_0 =	0.40 (p)	0.50 (p)	0.60 (p)	0.75 (p)	0.80 (p)	0.85 (p)	0.90 (p)
$z_t =$							
$\{\pi_{t-1}, x_t, x_{t-2}\}$	41.32 (0.00)	68.00 (0.00)	43.30 (0.00)	29.88 (0.00)	22.44 (0.00)	11.29 (0.00)	2.36 (0.12)
$\{\pi_{t-1}, x_t, \ldots, x_{t-2}\}$	52.72 (0.00)	51.46 (0.00)	47.71 (0.00)	35.06 (0.00)	30.21 (0.00)	26.86 (0.00)	17.27 (0.00)
$\{\pi_{t-1}, , x_{t-1}, x_{t-2}\}$	45.59 (0.00)	33.76 (0.00)	46.63 (0.00)	6.71 (0.01)	0.09 (0.77)	4.92 (0.03)	16.34 (0.00)
ω_0 =	0.075 (p)	0.15 (p)	0.25 (p)	0.35 (p)	0.45 (p)	0.50 (p)	0.60 (p)
$\{\pi_{t-1}, x_t, x_{t-2}\}$	6.96 (0.01)	5.01 (0.03)	1.18 (0.28)	0.26 (0.61)	0.01 (0.90)	0.16 (0.69)	1.34 (0.25)
$\{\pi_{t-1}, x_t, \ldots, x_{t-2}\}$	24.80 (0.00)	35.10 (0.00)	15.71 (0.00)	30.27 (0.00)	28.84 (0.00)	28.04 (0.00)	28.87 (0.00)
$\{\pi_{t-1}, x_{t-1}, x_{t-2}\}$	6.27 (0.01)	7.63 (0.01)	8.57 (0.00)	10.82 (0.00)	19.37 (0.00)	22.80 (0.00)	24.33 (0.00)

The Guggenberger–Smith *LM* statistic tests the null that $\theta = \theta_0$ or $\omega = \omega_0$. See Guggenberger and Smith (2008) for details.

A comparison of the first row of Table III (AR tests) with the first row of Table IV (GS tests) shows very different p-values, with the same instrument set. That difference occurs because the two tables test at different points in the parameter space. Table III fixes γ_f, which equals $\beta\theta/\phi$, to compute the AR statistics. Table IV fixes θ to compute the GS statistics. Thus, the AR statistic fixes the ratio/combination of three deep structural parameters, while the GS statistic sets θ conditional on being able to identify β and ϕ. The values of γ_f that are accepted in the first row of Table III (though rejected with more instruments later on in the same table) turn out, in Table IV, to be inconsistent with reasonable values of θ.

5.3. United Kingdom

The estimation sample for the UK is $1961Q1$–$2000Q4$. Table I shows that the Granger-causality pre-test provides strong evidence of predictability in both directions. This result implies that lagged values of inflation (beyond the first two lags) may be available as instruments. The second set of pre-tests indicates a lag length in the x-autoregression of $J = 5$ using the LR test and SIC. This places more of the history of x in the instrument vector z_t than in the US case.

Table V contains estimates of the UK hybrid NKPC. The GMM estimates depend on instrument choice. Once lags up to x_{t-4} are included, the coefficients accord with theory and are estimated with some precision. However, the over-identifying restrictions are rejected when x_t is an instrument. When we use only lagged instruments, the estimates of γ_f, γ_b, and λ are significant at the 10% level or better, and the p-value for the J-test rises to 0.22. Neiss and Nelson (2005) obtained statistically significant estimates of λ, but used dummy variables to control for a variety of price shocks. Table V also shows that the Calvo parameters are quite sensitive to the instrument set, a potential sign of weak identification.

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Table V. UK new Keynesian Phillips curve

$$E[\pi_t - \gamma_f E_t \pi_{t+1} - \gamma_b \pi_{t-1} - \lambda x_t | z_t]$$

1961Q1–2000Q4 $T = 168$

Instruments	$\hat{\gamma}_f$ (SE)	$\hat{\gamma}_b$ (SE)	$\hat{\lambda}$ (SE)	$\hat{\omega}$ (SE)	$\hat{\theta}$ (SE)	$\hat{\beta}$ (SE)	χ^2 (d.f.) (p)
π_{t-1}, x_t, x_{t-1}	−2.699	2.396	0.924	0.702	0.492	−1.608	—
	(4.782)	(3.047)	(1.531)	(0.113)	(0.337)	(1.412)	
$\pi_{t-1}, x_{t-1}, \ldots, x_{t-4}$	0.935	0.019	0.334	0.011	0.570	0.953	4.40(2)
	(0.266)	(0.192)	(0.152)	(0.114)	(0.065)	(0.113)	(0.22)
$\pi_{t-1}, x_t, \ldots, x_{t-4}$	0.234	0.535	0.062	0.562	0.801	0.307	9.82(3)
	(0.200)	(0.120)	(0.133)	(0.139)	(0.303)	(0.287)	(0.04)
$\pi_{t-1}, \pi_{t-2}, x_t, \ldots, x_{t-4}$	0.233	0.621	−0.045	0.842	1.569	0.201	15.94(4)
	(0.153)	(0.107)	(0.089)	(0.224)	(2.120)	(0.242)	(0.01)
OLS	0.413	0.400	0.107	0.393	0.708	0.574	
	(0.075)	(0.073)	(0.104)	(0.074)	(0.041)	(0.063)	

The estimation sample runs from 1961Q1 to 2000Q4, based on the complete 1959Q3–2001Q2 sample. Tests of the over-identifying restrictions use the J-statistic.

In contrast with the standard methods in Table V, however, Table VI gives evidence against the null of $\gamma_f = \gamma_{f0}$ for the UK hybrid NKPC. The significance levels of the AR statistics average 0.03 in Table VI, for the projection (19), on the same grid of values of γ_{f0} used for Table III. Only two of the 16 AR statistics have p-values that exceed 10%, which are associated with the instrument x_{t-1}, and γ_{f0} near unity. Once we include several instruments, in the second row of Table V, we find an asymptotic 95% confidence interval that is empty, as was the case for the USA. Again this is evidence against the hybrid NKPC.

Table VII presents the GS test statistics. The results are even more negative than in the US case shown in Table IV. The test rejects each value of θ or ω that we investigate, with each instrument set. Overall, then, the methods that are robust to weak identification provide a different impression of the UK NKPC than do standard methods.

Table VI. UK NKPC: Tests of $H_0 : \gamma_f = \gamma_{f0}$

$$\pi_t - \gamma_{f0}\pi_{t+1} = \alpha_0 + \alpha_1 \pi_{t-1} + \alpha_2 x_t + \alpha_3 u_t$$

Anderson–Rubin statistic

1961Q1–2000Q4 $T = 168$

$\gamma_{f0} =$	0.00 (p)	0.20 (p)	0.50 (p)	0.60 (p)	0.70 (p)	0.80 (p)	0.90 (p)	0.99 (p)
$u_t =$								
$\{x_{t-1}\}$	6.84 (0.01)	6.53 (0.01)	5.00 (0.03)	4.32 (0.04)	3.63 (0.06)	2.98 (0.09)	2.40 (0.12)	1.94 (0.17)
$\{x_{t-1}, \ldots, x_{t-4}\}$	4.52 (0.00)	4.58 (0.00)	4.53 (0.00)	4.47 (0.00)	4.40 (0.00)	4.32 (0.00)	4.24 (0.00)	4.18 (0.00)

See the notes to Tables III and V.

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Table VII. UK NKPC: Tests of $H_0 : \theta = \theta_0$ **or** $H_0 : \omega = \omega_0$

$$E[\gamma_f \pi_{t+1} - \pi_t + \gamma_b \pi_{t-1} + \lambda x_t | z_t] = 0,$$

$$\gamma_f = \frac{\beta\theta}{\phi}, \gamma_b = \frac{\omega}{\phi}, \lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\phi}, \phi = \theta + \omega[1-\theta(1-\beta)]$$

Guggenberger–Smith LM statistic $\sim \chi^2(1)$

1961Q1–2000Q4 $T = 168$

$\theta_0 =$	0.40 (p)	0.50 (p)	0.55 (p)	0.60 (p)	0.75 (p)	0.80 (p)	0.85 (p)
$z_t =$							
$\{\pi_{t-1}, x_t, x_{t-1}\}$	24.12 (0.00)	20.41 (0.00)	14.58 (0.00)	11.90 (0.00)	9.82 (0.00)	9.75 (0.00)	9.51 (0.00)
$\{\pi_{t-1}, x_t, \ldots, x_{t-4}\}$	30.83 (0.00)	28.33 (0.00)	28.37 (0.00)	27.66 (0.00)	26.98 (0.00)	22.14 (0.00)	19.43 (0.00)
$\{\pi_{t-1}, x_{t-1}, \ldots, x_{t-4}\}$	25.24 (0.00)	21.89 (0.00)	19.34 (0.00)	15.87 (0.00)	14.16 (0.00)	13.29 (0.00)	13.24 (0.00)
$\omega_0 =$	0.05 (p)	0.25 (p)	0.40 (p)	0.50 (p)	0.55 (p)	0.65 (p)	0.70 (p)
$\{\pi_{t-1}, x_t, x_{t-1}\}$	9.33 (0.00)	8.77 (0.00)	9.51 (0.00)	9.61 (0.00)	9.35 (0.00)	9.02 (0.00)	9.08 (0.00)
$\{\pi_{t-1}, x_t, \ldots, x_{t-4}\}$	39.73 (0.00)	42.22 (0.00)	36.12 (0.00)	26.15 (0.00)	39.04 (0.00)	37.12 (0.00)	38.75 (0.00)
$\{\pi_{t-1}, x_{t-1}, \ldots, x_{t-4}\}$	18.44 (0.00)	19.99 (0.00)	17.25 (0.00)	34.97 (0.00)	36.07 (0.00)	34.21 (0.00)	37.81 (0.00)

See the notes to Tables IV and V.

5.4. Canada

Estimating and testing the Canadian NKPC uses data from 1963Q1 to 2000Q4. Table I shows that Canadian inflation Granger-causes real marginal cost. This table also shows real marginal cost fails to Granger-cause inflation—in contrast to results for the UK and US data. The pre-tests for lag length reveal a persistence pattern similar to that in US real marginal cost, according to the LR test, the AIC, and the SIC. In the time series $\{x_t\}$, once-lagged costs play a large predictive role and thrice-lagged costs play an additional role that is statistically significant. However, a half-life of 8.5 quarters with respect to a shock to its third-order, autoregressive process shows that Canadian real marginal cost is more persistent than it is in the UK and the US data.

Table VIII contains estimates of the hybrid NKPC parameters γ_f, γ_b, and λ for Canada. They suggest that the hybrid NKPC is poorly identified. For example, the point estimates $\hat{\gamma}_f$ and $\hat{\gamma}_b$ are sensitive to the instrument set. When we include π_{t-2} as an instrument, these two coefficients are similar to those found in the US data, with a large role for future inflation. Guay *et al.* (2003) estimate the hybrid NKPC using a wider range of instruments. They increase precision and reject the over-identifying restrictions. However, we reproduce their finding that $\hat{\lambda}$ is insignificant. This indicates little role for real marginal cost in Canadian inflation dynamics. Table VIII also shows that either we cannot find an economically plausible value for the discount factor β or we obtain a wide 95% asymptotic confidence interval for β that runs from 0.71 to 1.24 (using the largest instrument vector).

Table IX includes AR statistics that favor forward-looking inflation dynamics for Canada, which is the opposite of the results found in the US and the UK data. None of the hypothesized values of γ_f can be rejected at the 5% level in Table IX. Thus, the AR 95% asymptotic confidence intervals for γ_f, $C(0.05)$, should cover plausible values. We find that these intervals are unbounded for each instrument set in Table IX.

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Table VIII. Canadian new Keynesian Phillips curve

$$E[\pi_t - \gamma_f E_t \pi_{t+1} - \gamma_b \pi_{t-1} - \lambda x_t | z_t] = 0$$

$1963Q1-2000Q4 \quad T = 152$

Instruments	$\hat{\gamma}_f$ (SE)	$\hat{\gamma}_b$ (SE)	$\hat{\lambda}$ (SE)	$\hat{\omega}$ (SE)	$\hat{\theta}$ (SE)	$\hat{\beta}$ (SE)	χ^2 (d.f.) (p)
π_{t-1}, x_t, x_{t-2}	−0.197	0.868	0.039	0.736	0.892	−0.188	—
	(2.085)	(1.374)	(0.074)	(0.186)	(0.040)	(1.726)	
$\pi_{t-1}, x_t, \ldots, x_{t-2}$	0.277	0.562	0.021	0.663	0.891	0.366	0.29(1)
	(0.768)	(0.514)	(0.027)	(0.287)	(0.033)	(1.201)	(0.86)
$\pi_{t-1}, x_{t-1}, \ldots, x_{t-4}$	−1.052	1.466	0.061	0.785	0.902	−0.625	1.25(3)
	(1.274)	(0.876)	(0.049)	(0.061)	(0.039)	(0.405)	(0.87)
$\pi_{t-1}, \pi_{t-2}, x_{t-1}, \ldots, x_{t-4}$	0.716	0.274	0.005	0.341	0.911	0.979	2.48(4)
	(0.167)	(0.121)	(0.009)	(0.191)	(0.053)	(0.133)	(0.78)
OLS	0.442	0.430	0.019	0.534	0.889	0.618	
	(0.047)	(0.043)	(0.011)	(0.029)	(0.017)	(0.013)	

The estimation sample is $1963Q1-2000Q4$ with leads and lags taken from a $1961Q1-2001Q1$ sample. Tests of the over-identifying restrictions use the J-statistic.

Table IX. Canadian NKPC: Tests of $H_0: \quad \gamma_f = \gamma_{f0}$

$$\pi_t - \gamma_{f0}\pi_{t+1} = \alpha_0 + \alpha_1 \pi_{t-1} + \alpha_2 x_t + \alpha_3 u_t$$

Anderson–Rubin statistic

$1963Q1-2000Q4 \quad T = 152$

γ_{f0} =	0.00 (p)	0.20 (p)	0.50 (p)	0.60 (p)	0.70 (p)	0.80 (p)	0.90 (p)	0.99 (p)
$u_t =$								
$\{x_{t-2}\}$	0.01 (0.91)	0.07 (0.80)	0.22 (0.64)	0.28 (0.60)	0.33 (0.57)	0.37 (0.54)	0.41 (0.52)	0.43 (0.51)
$\{x_{t-1}, x_{t-2}\}$	0.40 (0.67)	0.31 (0.74)	0.20 (0.82)	0.18 (0.84)	0.18 (0.84)	0.19 (0.83)	0.20 (0.82)	0.22 (0.80)
$\{x_{t-1}, \ldots, x_{t-4}\}$	0.69 (0.60)	0.82 (0.52)	0.95 (0.44)	0.96 (0.43)	0.96 (0.43)	0.94 (0.44)	0.91 (0.46)	0.87 (0.48)

See the notes to Tables III and VIII.

The non-rejection (and unbounded confidence intervals) in Table IX may reflect a lack of test power. Table X presents the GS test statistics for Canada. Recall that this test has greater power in over-identified cases. With relatively small instrument sets, there is some support for the selected values of θ or ω, but at these values the estimates of γ_f are less than 0.1. But with larger instrument sets, all the values of θ and ω we consider are rejected. Thus, the GS tests provide very limited support for the hybrid NKPC in Canadian data.

5.5. Discussion

Overall, the finding from tests robust to weak identification is that there is little evidence of forward-looking dynamics in US, UK, and Canadian inflation. Moreover, based on estimates of λ—the slope of the NKPC—there also is little evidence of a significant relationship between inflation and marginal cost in these three countries and time periods. Point estimates $\hat{\lambda}$ generally

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Table X. Canadian NKPC: Tests of $H_0: \theta = \theta_0$ or $H_0: \omega = \omega_0$

$$E[\gamma_f \pi_{t+1} - \pi_t + \gamma_b \pi_{t-1} + \lambda x_t | z_t] = 0,$$

$$\gamma_f = \frac{\beta\theta}{\phi}, \quad \gamma_b = \frac{\omega}{\phi}, \quad \lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\phi}, \quad \phi = \theta + \omega[1-\theta(1-\beta)]$$

Guggenberger–Smith LM statistic $\sim \chi^2(1)$

$1963Q1–2000Q4 \quad T = 152$

θ_0 =	0.40 (p)	0.50 (p)	0.60 (p)	0.75 (p)	0.80 (p)	0.85 (p)	0.90 (p)
z_t =							
$\{\pi_{t-1}, x_t, x_{t-2}\}$	2.14 (0.14)	4.63 (0.03)	11.81 (0.00)	26.09 (0.00)	28.53 (0.00)	8.49 (0.00)	0.06 (0.81)
$\{\pi_{t-1}, x_t, \ldots, x_{t-2}\}$	7.33 (0.01)	7.41 (0.01)	7.90 (0.00)	4.49 (0.03)	14.08 (0.00)	19.25 (0.00)	2.51 (0.11)
$\{\pi_{t-1}, x_{t-1}, \ldots, x_{t-4}\}$	39.66 (0.00)	41.13 (0.00)	46.81 (0.00)	28.59 (0.00)	18.42 (0.00)	9.42 (0.00)	3.65 (0.06)

ω_0 =	0.40 (p)	0.50 (p)	0.60 (p)	0.65 (p)	0.70 (p)	0.75 (p)	0.80 (p)
$\{\pi_{t-1}, x_t, x_{t-2}\}$	8.40 (0.00)	6.86 (0.01)	3.46 (0.06)	1.57 (0.21)	0.33 (0.56)	0.03 (0.86)	1.32 (0.25)
$\{\pi_{t-1}, x_t, \ldots, x_{t-2}\}$	6.54 (0.01)	4.75 (0.03)	2.88 (0.09)	2.41 (0.12)	3.16 (0.08)	6.54 (0.01)	14.46 (0.00)
$\{\pi_{t-1}, x_{t-1}, \ldots, x_{t-4}\}$	19.28 (0.00)	17.15 (0.00)	14.24 (0.00)	12.71 (0.00)	10.68 (0.00)	8.49 (0.00)	10.15 (0.00)

See the notes to Tables IV and VIII.

are positive but have large standard errors. Inspection of the graphs of $\{\pi_t, x_t\}$ in Figure 1 shows why it is hard to argue that π_t tracks x_t, whatever the dynamics, in these data.

This raises the question of why some other studies find greater evidence in favor of the new Keynesian Phillips curve. There are three main differences across studies. First, studies may vary in their measure of x_t, either by measuring marginal cost in different ways or by using an output gap. As we have noted, studies that use an output gap tend to find greater evidence of a positive slope, λ.

Second, a variety of studies we have cited make use of additional information such as survey data on expectations or restrictions on a system. For example, several studies in the September 2005 special issue of the *Journal of Monetary Economics* on the econometrics of the new Keynesian price equation take this approach rather than using the limited information estimators we have studied here.

Third, studies that do use limited information/GMM estimation may differ in their choice of instruments. Recall that one of the symptoms of weak identification is the use of large instrument sets and that under weak identification GMM estimates are biased in the direction of OLS estimates. In the last row of Tables II, V, and VIII we include OLS estimates of the NKPC as a limiting case. For the USA, Table II shows that OLS estimates naturally have smaller standard errors than the GMM estimates. They imply a smaller weight on expected future inflation and a larger weight on lagged inflation. But they still find an insignificant slope to the US Phillips curve. For the UK, Table V shows that the OLS estimates involve greater weight on future inflation, less weight on past inflation, and a larger, positive point estimate $\hat{\lambda}$ than do the previous two rows of GMM estimates. For Canada, Table VIII shows that the estimates are very sensitive to the instrument set, with the OLS values lying somewhere in the middle of the range of GMM estimates. Once again the standard errors of $\hat{\gamma}_f$ and $\hat{\gamma}_b$ are small and the point estimate $\hat{\lambda}$ is positive.

For all three countries, OLS yields a positive, significant weight, $\hat{\gamma}_f$, on future inflation. The OLS results provide a limiting case of GMM as weak instruments are added. They contrast with the findings that are robust to weak identification where, especially for the USA and UK, there is little evidence of forward-looking inflation dynamics.

6. CONCLUSION

This paper is about identification problems in the hybrid new Keynesian Phillips curve (NKPC). We show that estimation of the hybrid NKPC by GMM faces a fundamental source of non-identification: weak, higher-order dynamics. By setting the hybrid NKPC in a new Keynesian, three-equation model, we find the hybrid NKPC cannot be identified by GMM even when shocks are persistent.

We draw on the Anderson–Rubin statistic to provide a new set of tests of the forward-looking inflation model. These test statistics are robust to either weak or omitted instruments. The tests reveal little evidence of forward-looking expectations driving US or UK inflation. When we add to test power by using the tests derived by Guggenberger and Smith (2008), we also reject the model of inflation for Canada. It is noteworthy that in all three cases the conventional J-test statistic does not reject the over-identifying restrictions.

This study has used asymptotic critical values. It would also be worthwhile to study the sampling properties of the estimators and test statistics in an economically interesting DGP. As Section 4 shows, the NKTM probably does not qualify, at least for the study of the limited information methods used here.

Rejecting a necessary condition is sufficient for rejecting a model. As usual, then, an advantage of the limited information approach is that it may alert us to an empirical difficulty without requiring a complete model. But whether the test rejections here are due to misspecification of the economic model of inflation, or to measurement problems, or to the assumption of rational expectations is worthy of further exploration.

APPENDIX A: ANALYTICAL SUMMARY AND PROOFS

Proposition 1 If a consistent estimate $\hat{\lambda}$ is available, then an additional instrument is available in z_t but not in z_{t-1}.

Proof: The rank condition requires three instruments to identify $\{\gamma_b, \gamma_f, \lambda\}$. When λ is known or estimated from auxiliary information then x_t becomes a valid instrument for π_{t+1}; the instruments x_t and π_{t-1} can be used to identify γ_f and γ_b. But with instruments z_{t-1} three variables in the NKPC (3) remain to be forecast, $\{\pi_{t+1}, \pi_t, x_t\}$, even given an estimate $\hat{\lambda}$. Thus, a two-step procedure does not aid identification with lagged instruments.

Proposition 2 Restricting $\gamma_b = 0$, or $\gamma_b = 1 - \gamma_f$, or calibrating a discount factor β in an underlying pricing model may aid identification.

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Proof: Any one of these restrictions reduces the number of parameters to be estimated from three to two, so that only two instruments now are needed for identification.

Proposition 3 Solving the NKPC forward and truncating provides no additional information to aid identification (or improve efficiency).

Proof: The difference equation—solved forward and truncated—still involves the three parameters $\{\gamma_f, \gamma_b, \lambda\}$. Were there valid instruments for each future x_{t+k} in the solution (4), these parameters would be over-identified because the estimating equations (7) contain more variables than parameters when $K \geq 1$. Nonetheless, the number of relevant instruments is unaffected by the transformation, so the conditions for identification are unchanged.

Proposition 4 Whether z_t or z_{t-1} is adopted, the GMM residual is an MA(1) process, so any instrument set must exclude once-lagged GMM residuals.

Proof: Suppose that x_t evolves as an autonomous, first-order autoregression:

$$x_t = \rho x_{t-1} + \varepsilon_t \tag{A1}$$

where ε_t is an innovation with respect to past values of x. This setup is without loss of generality, for higher-order dynamics also can be written in first-order, state-space form with a suitable definition of x. First, suppose that $x_t \in z_t$. Using well-known methods the solution to the present-value model (4) is

$$E[\pi_t|\pi_{t-1}, x_t] = \delta_1 \pi_{t-1} + \frac{\lambda}{(\delta_2 - \rho)\gamma_f} x_t \tag{A2}$$

Define the forecast error:

$$\eta_t = \pi_t - E[\pi_t|\pi_{t-1}, x_t] \tag{A3}$$

which arises because agents may have more information than the econometrician. From the NKPC (1) and (A1)–(A3), the estimating equation error is

$$\gamma_f(\pi_{t+1} - E[\pi_{t+1}|\pi_{t-1}, x_t]) - (\pi_t - E[\pi_t|\pi_{t-1}, x_t]) = \gamma_f \left(\delta_1 \eta_t + \eta_{t+1} + \frac{\lambda}{(\delta_2 - \rho)\gamma_f} \varepsilon_{t+1} \right) - \eta_t \tag{A4}$$

which follows an MA(1) process. Second, suppose that $x_t \notin z_t$ but that $x_{t-1} \in z_t$. The estimating equation error is

$$\begin{aligned} &\gamma_f(\pi_{t+1} - E[\pi_{t+1}|\pi_{t-1}, x_{t-1}]) - (\pi_t - E[\pi_t|\pi_{t-1}, x_{t-1}]) \\ &\quad + \lambda(x_t - E[x_t|\pi_{t-1}, x_{t-1}]) = \gamma_f \left(\eta_{t+1} + \delta_1 \eta_t + \frac{\lambda}{(\delta_2 - \rho)\gamma_f} \varepsilon_{t+1} \right. \\ &\quad \left. + (\delta_1 + \rho) \frac{\lambda}{(\delta_2 - \rho)\gamma_f} \varepsilon_t \right) - \left(\eta_t + \frac{\lambda}{(\delta_2 - \rho)\gamma_f} \varepsilon_t \right) + \lambda \varepsilon_t \end{aligned} \tag{A5}$$

which again is MA(1). In either case this composite error thus is not orthogonal to its lagged value, so use of the lagged value as an instrument violates the order condition.

Proposition 5 In the new Keynesian, three-equation model with unpredictable shocks, the hybrid NKPC cannot be identified by single-equation GMM.

Proof: In the first-order Markov solution (13), π_{t+1} can be predicted by $w_t = (\pi_t y_t)'$. With y_t entering the NKPC as a separate regressor, no instruments are available for π_{t+1}.

Proposition 6 In the NKTM (8) with persistent shocks the hybrid NKPC is not identified under GMM.

Proof: The proof proceeds in two steps. First, we show that the shocks to the solved model are correlated, even if the underlying shocks in the NKTM are not correlated. Suppose that the 2×1 vector of shocks, ε_t, follows a Jth-order autoregression. Stack the shocks like this:

$$\tilde{\varepsilon}_t = (\varepsilon_t \varepsilon_{t-1} \cdots \varepsilon_{t-J+1})' \tag{A6}$$

The autoregression can be written

$$\tilde{\varepsilon}_t = \Lambda \tilde{\varepsilon}_{t-1} + \vartheta_t \tag{A7}$$

where ϑ is a 2×1 vector of innovations and Λ is a matrix of size $2(J-1) \times 2(J-1)$. The solution to the NKTM is of the form

$$w_t = a w_{t-1} + b\tilde{\varepsilon}_t \tag{A8}$$

using the standard result that the number of lags in the solution is one less than the order of the autoregression. Combining (A7) and (A8) gives

$$[I - \Lambda L] w_t = a[I - \Lambda L] w_{t-1} + b\vartheta_t \tag{A9}$$

so that the dynamics of w_t are of order $J + 1$.

Solving for a and b by the methods of undetermined coefficients of Zadrozny (1998) involves a two-step process. First, note the endogenous bivariate autoregressive dynamics imply

$$\begin{pmatrix} c & 0 \\ 0 & I \end{pmatrix} \begin{bmatrix} E_t w_{t+1} \\ w_t \end{bmatrix} + \begin{pmatrix} -[I-f] & d \\ -I & 0 \end{pmatrix} \begin{bmatrix} w_t \\ w_{t-1} \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{A10}$$

The 4×4 matrix attached to $[E_t w_{t+1} w_t]'$ is nonsingular, so standard eigenvalue routines allow us to recover the four elements of a.

Given a solution for a, compute the four elements of b by substituting for $E_t w_{t+1}$ and w_t in (A10) employing the conjectured solution (A8). These actions give us

$$[(ca - [I-f])a + d] w_{t-1} = -cbE_t\varepsilon_{t+1} - [I + (ca - [I-f])b]\varepsilon_t \tag{A11}$$

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Next use the law of motion for the shocks (A7) to substitute for $E_t\varepsilon_{t+1}$ in (A11) and pass the *vec* operator through the result to obtain the solution

$$vec(b) = -[(\rho_\varepsilon' \otimes c) + (I' \otimes (ca - [I - f])]^{-1} vec(I) \tag{A12}$$

where use is made of the fact that $vec(JKL) = (L' \otimes J)vec(K)$. Thus, b is not, in general, a diagonal matrix given the structure of the matrices c, f, and a. Therefore the two elements of $b\vartheta_t$ are correlated.

Second, we show that this correlation rules out identification. To find candidate instruments for π_{t+1}, we use the common-factor model (A9) because it involves only observable variables and innovations. Future inflation π_{t+1} can be predicted using the $2 \times J$ set of variables $\{w_t, w_{t-1}, \cdots w_{t-J}\}$.

Valid instruments also must be uncorrelated with the residual in the NKPC, $\varepsilon_{\pi t}$. For any positive J and given the correlation between the two elements of $b\vartheta_t$, the variables x_{t-1} and y_t also must be instrumented in the NKPC. But ε_{π_t} is correlated with $\{w_t, w_{t-1}, \cdots w_{t-J}\}$, so there are no valid instruments.

Proposition 7 In the NKTM (8) with interest rate smoothing in monetary policy the hybrid NKPC is not identified under GMM.

Proof: Interest rate smoothing gives

$$R_t = (1 - \upsilon)(\omega_\pi \pi_t + \omega_y y_t) + \upsilon R_{t-1} + \varepsilon_{R_t} \tag{A13}$$

with $0 < \upsilon < 1$. Define the new state vector: $w_t = (\pi_t y_t R_t)'$. This remains first-order Markov, so π_{t+1} can be predicted only by w_t. Among the elements of w_t, π_t is ineligible as an instrument because it is the regressand in the NKPC, while y_t and R_t are correlated with $\varepsilon_{\pi t}$ because both the *IS* curve and the policy rule contain π_t.

Proposition 8 When the NKTM possesses multiple equilibria and the rational expectations forecast errors are a (linear) function of the fundamental and extrinsic shocks, the hybrid NKPC is not identified under GMM.

Proof: The bivariate ARMA model (17) has a moving average component with no roots inside the unit circle, and so the vector moving average is nonfundamental. The lag polynomial operator for the sunspot innovation is invertible only in the forward direction, as shown by Lippi and Reichlin (1994). Thus, π_{t+1} is not predicted by lagged, observable variables as a result of the sunspot, so no new instruments are available.

APPENDIX B: DATA SOURCES

United States

The price level P_t is the GDP implicit price deflator. The GDP deflator is available in chain weight form and in implicit form (all the US results are based on the implicit GDP deflator). Nominal unit labor cost (ULC) is the ratio of the index of hourly compensation in the non-farm business sector,

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labelled COMPNFB, to output per hour of all persons in the non-farm business sector, labelled OPHNFB. COMPNFB is an index of the nominal wage. OPHNFB is an index of the average product of labor. These can be found in the Federal Reserve Bank of St Louis FRED databank. Thus, ULC is a measure of labor's share. Real ULC equals nominal ULC deflated by P_t. Inflation is $100\ln(P_t/P_{t-1})$ and real ULC is $100(1+q)\ln(\text{COMPNFB}_t/\text{OPHNFB}_t) - 100\ln P_t$, where q is a function of the steady-state markup and labor's share parameter in the firm's production function. This adjustment renders real ULC stationary and $q = 1.08$. The entire sample runs from 1947Q1 to 2002Q4, but the estimation sample period is 1949Q1–2001Q4, $T = 212$.

United Kingdom

The inflation rate is measured with the GDP deflator, and x is a measure of the log of real marginal cost. Data sources are given by Neiss and Nelson (2005), who kindly provided the data. The estimation period is 1961Q1–2000Q4, so $T = 168$.

Canada

The inflation rate is measured with the GDP deflator, while x is the log of the labour share in the non-farm, business sector. Data sources are given by Guay *et al.* (2003), who kindly provided the data. The estimation period is 1963Q1–2000Q4.

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